647510 **BSc4MatC0401** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination
March/April-2023 [OLD COURSE]
MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any One) (07)

1. If V is a set of all real valued continuous (differentiable or integrable) functions of x defined on some interval [0,1]. For $f, g \in V;\ \alpha \in R$, $(f+g)(x) = f(x) + g(x), (\alpha f)(x) = \alpha f(x)$ then show that V is vector space.
2. If vector $\bar{v}_k (1 \le k \le n)$ of set $(\bar{v}_1, \bar{v}_2 \dots \bar{v}_n)$ is linearly combination of remaining vectors $\bar{v}_1, \bar{v}_2 \dots \bar{v}_{k-1}, \bar{v}_{k+1}, \dots \bar{v}_n$ then $Sp\{\bar{v}_1, \bar{v}_2 \dots \bar{v}_n\} = Sp\{\bar{v}_1, \bar{v}_2 \dots \bar{v}_{k-1}, \bar{v}_{k+1}, \dots \bar{v}_n\}$.

Q.1(B) Answer the following question. (Any One) (04)

1. Show that $W = \{(x, y, z) \in R^3 / \ x - 2y + 3z = 0\}$ is subspace of R^3.
2. Define: Linearly dependent set of vectors. Show that $A = \{(1,0,1), (1,1,0), (-1,0,-1)\}$ is linearly dependent.

Q.1(C) Answer the following question. (Any One) (03)

1. Define: Vector space.
2. Check whether $(1,2,4), (1,5,4) \in SpA$ where $A = \{(0,1,-1), (0,0,2), (1,3,0)\}$.

Q.2(A) Answer the following questions. (Any One) (07)

1. Prove that there exists a basis for each finite dimensional vector space V.
2. If W_1 and W_2 are two subspaces of a finite dimensional vector space V then prove that $dim(W_1 + W_2) = dimW_1 + dimW_2 - dim(W_1 \cap W_2)$ and hence deduce that if $V = W_1 \oplus W_2$ then $dim\text{V} = dimW_1 + dimW_2$.

Q.2(B) Answer the following question. (Any One) (04)

1. Show that $B = \{(2,2,3), (2,1,3), (1,0,1)\}$ is basis of R^3. Find co-ordinates of vector (1,1,1) with respect to B.
2. Expand set $A = \{(1,1,1), (2,0,0)\}$ of vector space R^3 to form a basis of R^3.

Q.2(C) Answer the following question. (Any One) (03)

1. If $\bar{u} = (x_1, y_1, z_1), \bar{v} = (x_2, y_2, z_2), \bar{w} = (x_3, y_3, z_3)$ are linearly dependent then show that: $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$.
2. Define: **(i)** Basis of vector space **(ii)** dimension of vector space

Q.3(A) Answer the following questions. (Any One) (07)

1. Define: Range and kernel of linear transformation. Prove that: A linear transformation $T: U \to V$ is one-one iff $N_T = \{\theta\}$.
2. State and prove rank-nullity theorem for linear transformation.

Q.3(B) Answer the following question. (Any One) (04)

1. Show that $T: R^4 \to R^4, T(a,b,c,d) = (a-b, b-c, c-d, d)$ is non singular and find T^{-1}.
2. Prove that composition of two linear transformation is again a linear transformation.

Q.3(C) Answer the following question. (Any One) (03)

1. If $T: R^2 \to R^2, T(x,y) = (x+1, y-2), \forall (x,y) \in R^2$ then show that T is not linear transformation.
2. If $S, T: R^3 \to R^3, S(a,b,c) = (a-3b, b+5c)$ and $T(a,b,c) = (7a, b-2c), \forall (a,b,c) \in R^3$ are linear transformations then find $S+T, 5S, 7T$.

Q.4(A) Answer the following questions. (Any One) (07)

1. Let $T; V \to V$ be a linear transformation and B be any basis of V. Prove: T is singular iff $det([T; B]) = 0$.
2. Find eigen values and eigen vectors for the linear transformation $T: R^3 \to R^3$ defined as $T(x,y,z) = (-2y-2z, -2x-3y, 3x+6y+5z), \forall (x,y,z) \in R^3$ by considering the standard basis of R^3.

Q.4(B) Answer the following question. (Any One) (04)

1. If $T: R^2 \to R^2, T(x.y) = (x, -y), \forall (x,y) \in R^2$ and $B_1 = \{(1,1),(1,0)\}, B_2 = \{(2,3),(4,5)\}$ then find $[T; B_1, B_2]$.
2. Let $T; V \to V$ be a linear transformation and B be a basis of V. Prove: λ is eigen value of T iff $det([(T - \lambda I_n),\ B]) = 0$, where $I_n: V \to V$ is the identity linear transformation on V and $\dim(V) = n$.

Q.4(C) Answer the following question. (Any One) (03)

1. Define: matrix associated with linear transformation.
2. Define: **(i)** Dual of a vector space **(ii)** Adjoint of a linear transformation.

Q.5(A) Answer the following questions. (Any One) (07)

1. Prove that radius of curvature of the curve $r = f(\theta)$ at $P(r,\theta)$ is
   $\rho = \frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2 - rr_2 + 2{r_1}^2}$ where $r^2 - rr_2 + 2{r_1}^2 \neq 0$.
2. Define: Oblique asymptote. If $y = mx + c$ is an asymptote to the curve $y = f(x)$, prove that
   $m = \lim_{x \to \pm\infty} \left(\frac{y}{x}\right)$ and $c = \lim_{x \to \pm\infty} (y - mx)$.

Q.5(B) Answer the following question. (Any One) (04)

1. Find multiple points of the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine its type.
2. Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.

Q.5(C) Answer the following question. (Any One) (03)

1. Prove that $y = logx$ is concave upwards everywhere.
2. Define: Curvature and radius of curvature.

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